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Experimental study of the electron collision effects on the resonance cone phenomenon

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This Rapid Communication deals with the electron-density measurements using the resonance cone phenomenon in a cold magnetized plasma. The collision frequency limits the modulus of the upper hybrid resonance cone peak and shifts it. Theoretically, this effect is slight and should not affect the electron-density measurements done with this method in magnetized plasmas. The experiment reported in this paper shows that the phenomenological collision frequency introduced in the theoretical potential is much higher than expected on the basis of electron-ion and electron-neutral collisions, so that it is necessary to examine not only the amplitude of the signal but also its phase, in order to estimate this phenomenological frequency and thus the correction to bring about the electron density. [S1063-651X(98)50207-3]

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The study of the resonance cone phenomenon near the upper hybrid frequency of a magnetized plasma started in the 1950's [1,2]. These resonance cones have been observed experimentally first by Fisher and Gould [3] in the late 1960's and then in the 1970's by Gonfalone [4]. Among others, Kuehl [5] studied the phenomenon concerning the theoretical point of view. In particular, the nonphysical aspect of the emitted potential singularity has been explained by the thermal effects or by collisions. If numerous publications have treated the thermal effects that generate according to some authors, an ''interference pattern'' [6], surprisingly few studies have been done on the effect of collisions on resonance cone phenomenon [7]. Yet collision effects are of the greatest importance [8]; indeed, the collisions limit the amplitude of the singularity, but also introduce a shift in the position of the cone, so that the measurement of the electron density is affected. In this paper we present an experimental study of this effect. After we briefly recall some theoretical results on resonance cones and show the effect of collisions, experimental results are compared with the theoretical model in a physical situation where collisions have to be taken into account. In conclusion, we propose an experimental procedure to measure electron density by this method [9].

It is well known that a wave propagating in a magnetized plasma presents a resonance for an angle θ that verifies $\tan^2\theta=-\varepsilon_\parallel/\varepsilon_\perp$, where $\varepsilon_\parallel$ and $\varepsilon_\perp$ represent, respectively, the dielectric tensor elements parallel and perpendicular to the magnetic field [10,12,13]. This is the ''phase-velocity (v_ϕ) resonance cone'' since in this case the refractive index $\vec{N}=c\vec{k}/\omega\rightarrow\infty$, that is to say, $v_\phi\rightarrow 0$, where c is the speed of light in vacuum, $\vec{k}$ is the wave vector, and ω is the wave angular frequency. There is another resonance cone called ''group-velocity (v_g) resonance cone'' (usually called simply ''resonance cone'') for which $v_g\rightarrow 0$. This one is obviously the only one that can be experimentally observed. If the collision frequency is not taken into account, the cone angle κ verifies $\tan^2\kappa=-\varepsilon_\perp/\varepsilon_\parallel$, which is commonly written in the form

$$\sin^2\kappa=\frac{\omega^2(\omega_{\mathrm{pe}}^2+\omega_{\mathrm{ce}}^2-\omega^2)}{\omega_{\mathrm{pe}}^2\,\omega_{\mathrm{ce}}^2},\tag{1}$$

where ω_{pe} and ω_{ce} are, respectively, the electron-plasma and the electron-cyclotron angular frequencies. Thus, $\kappa+\theta=\pi/2$.

The studies of thermal effects on resonance cones show that the cone peak is slightly shifted with respect to the so-called cold-plasma cone (that is to say, without thermal effects). But it is known that a cold plasma could be highly collisional, so the collisional effects cannot be neglected. We introduce into the theory a phenomenological collision frequency ν in the dielectric tensor. The derivation of the potential Φ of the resonance cone, taking into account collisions, is described in [8] as

$$\Phi(\rho,z)=\frac{q_e}{8\pi\varepsilon_\perp}\frac{1}{[z^2+\rho^2(\varepsilon_\parallel/\varepsilon_\perp)]^{1/2}},\tag{2}$$

where q_e is the elementary electric charge, and z and ρ are, respectively, the coordinates perpendicular and parallel to $\vec{B}_0$. The cone angle κ_c, taking into account phenomenological collision frequency ν, now verifies that $\tan^2\kappa_c=-\mathrm{Re}(\varepsilon_\parallel/\varepsilon_\perp)$, where Re represents the real part of the complex number. Figures 1 and 2 represent the different positions of the cone peak (amplitude and phase, respectively) for different ν values. The peak shift is slight (less than 1%) for small $\nu_r=\nu/\omega_{\mathrm{ce}}$ values (less than $10^{-3}\ \mathrm{s}^{-1}$) for the calculation parameters. We have done experiments on a magnetized plasma, for different theoretical ν values, assuming that collisions are mainly due to electron-ion collisions:

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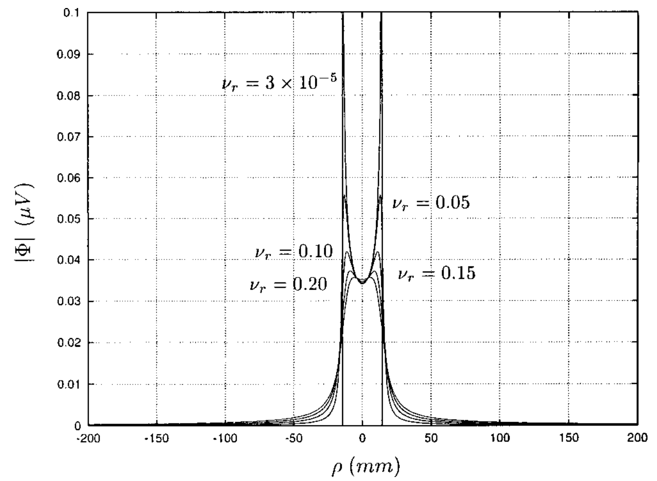

FIG. 1. Computed potential Φ modulus for different $\nu_r = \nu/\omega_{ce}$ values for $\omega_{per} = \omega_{pe}/\omega_{ce} = 0.85$ and $\omega_r = \omega/\omega_{ce} = 0.60$.

$$\nu = \nu_{ei} = \frac{\sqrt{2}\ Z n_e\ e^4\ \ln\Lambda}{12\pi^{3/2}\varepsilon_0^2\sqrt{m_e}(k_B T_e)^{3/2}},$$

where n_e is the electron density, T_e is the electron temperature, $Z \approx 10$ is the average ion-charge number, and $\ln\Lambda \approx 12$ is the Coulombian logarithm. We neglected the electron scattering on a wave. The collision frequency given by this phenomenon is, after [14],

$$\nu_{ew} = 3.5\frac{n_e\ v}{k^2}\left(\frac{e\Phi}{k_B T_e}\right)^2,$$

where v is the electron velocity and k is the wave number. Since the potential is low ($\Phi < 10^{-6}$ V) the electron velocity is close to the thermal velocity. Near the resonance cone, we have $k \to \infty$ so that $\nu_{ew} \to 0$; the electron-wave collision frequency is negligible with respect to the electron-ion collision frequency near the resonance cone.

The device described in Fig. 3 consists of a vacuum chamber in which a quasi-homogeneous magnetic field $\vec{B}_0$ is created with two Helmholtz coils. The chamber is filled with argon gas at a pressure of $p_n \approx 5 \times 10^{-4}$ mbar. This corre-

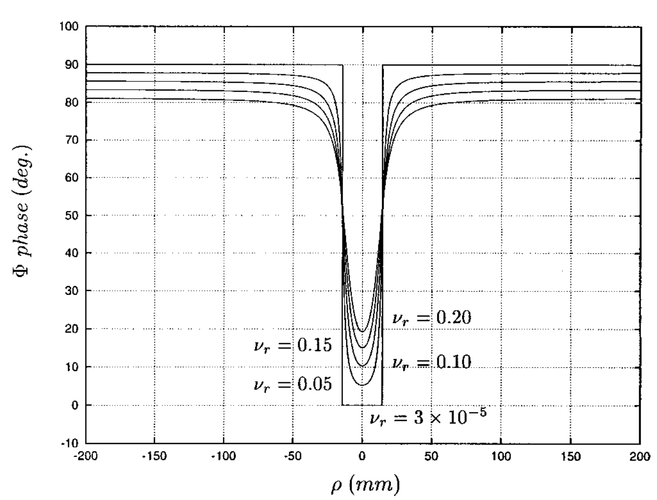

FIG. 2. Computed potential Φ phase for different $\nu_r = \nu/\omega_{ce}$ values for $\omega_{per} = \omega_{pe}/\omega_{ce} = 0.85$ and $\omega_r = \omega/\omega_{ce} = 0.60$.

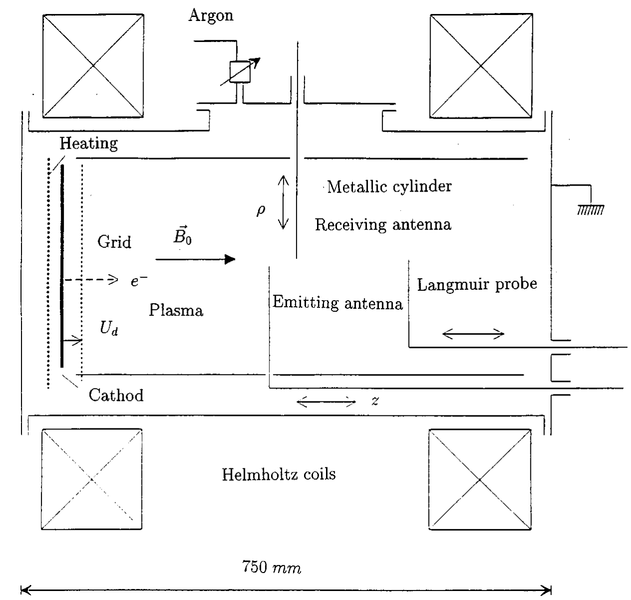

FIG. 3. Schematic diagram of the LPGP-Orsay apparatus.

sponds to $k_B T_n \approx 0.08$ eV and $n_n \approx 4 \times 10^{12}$ cm^{-3}. The plasma is created by heating a cathode covered with barium, calcium, and strontium oxide; the accelerated electrons produced a discharge that can work in a continuous or a pulsed mode. The range of parameters that can be obtained in the continuous mode that we are using are of the order of 10^9 cm$^{-3} < n_e < 10^{11}$ cm^{-3} and 1 eV$< k_B T_e <$4 eV [11]. An antenna immersed in the quasihomogeneous plasma emits a wave; the wave detection is made by an antenna moving along z and motorized along ρ. The antennae are made of tungsten wire that is 0.2 mm in diameter and 2 mm in length. From the detected signal, the ac part is only selected with a dc block. Two amplifiers and then a crystal detector (quadratic detector) are used; the latter give the square of the received signal amplitude. The signal is then monitored using a numerical oscilloscope. To measure the phase of the signal, a mixer is used instead of the crystal detector, which can measure the phase difference between the emitted signal and the received signal. All of the experiments were performed with $B_0 \approx 120$ G, $n_e \approx 10^9$ cm^{-3}, $k_B T_e \approx 3$ eV, and $f = 200$ MHz; that is to say, $\omega_{per} = \omega_{pe}/\omega_{ce} \approx 0.85$ and $\omega_r = \omega/\omega_{ce} \approx 0.60$.

The measured signals for different positions on the z axis (10 mm, 30 mm, and 50 mm) are presented in Fig. 4 for the amplitude and in Fig. 5 for the phase. There is a gap in the phase curve between the two amplitude curve peaks as was expected. The thermal effects are not observed; this is probably due to the plasma noise and the collision frequency, as will be explained further. In Fig. 6 the signal phase for $z = 10$ mm is shown. But this gap does not correspond exactly to the one simulated in our model, for the same plasma parameters, as we show in Fig. 5 for the amplitude and in Fig. 6 for the phase. Setting the phenomenological collision frequency to higher values in the theoretical model, the resulting curves are given in Figs. 1 and 2. Then the experimental curves can be compared to the theoretical curves. Indeed, it appears that the real phenomenological collision frequency ν value is much higher than the one expected from the plasma

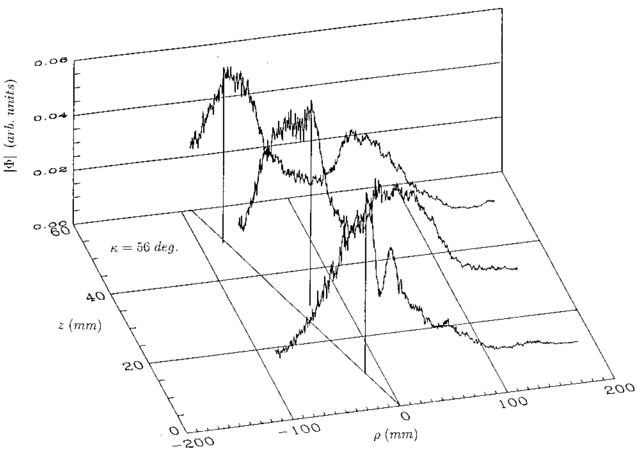

FIG. 4. Measured signal modulus for different z values ($z = 10$, 20, and 30 mm), and for $B_0 = 120$ G, $f = 200$ MHz, $p_n \approx 5 \times 10^{-4}$ mbar, and discharge current $I_d = 0.46$ A.

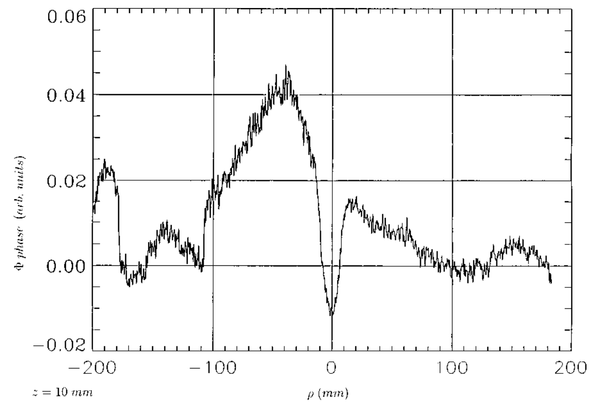

FIG. 6. Measured signal phase for $z = 10$ mm and $B_0 = 120$ G, $f = 200$ MHz, $p_n \approx 5 \times 10^{-4}$ mbar and discharge current $I_d = 0.46$ A.

parameters ($\nu_r \approx 3 \times 10^{-5}$). Accordingly, the actual collision frequency is around 5×10^3 times larger than the expected one. This discrepancy can be explained by wave turbulence in plasma [15].

This high phenomenological collision frequency ν introduces a strong shift of the resonance peak cone and, therefore, a change in the electron-density determination, as seen in Fig. 1. Using Eq. (1), we described the relationship between the variation on n_e, Δn_e, and the variation on the peak position κ, $\Delta\kappa = \kappa' - \kappa$ with κ' as the cone angle, which would be obtained for a collisionless plasma, by the following relation:

$$\frac{\Delta n_e}{n_e} = \frac{\omega_{\rm ce}^2 \sin 2\kappa}{\omega_{\rm ce}^2 \sin^2\kappa - \omega^2} \times \Delta\kappa, \tag{3}$$

with $\Delta n_e = n_e' - n_e$, where n_e' would be the electron-density value inferred from the measurement if it were collisionless.

The results on relative correction about electron density are presented in Fig. 7 versus $\omega_{\rm per} = \omega_{\rm pe}/\omega_{\rm ce}$ for different $\nu_r = \nu/\omega_{\rm ce}$ values, and for $\omega_r = \omega/\omega_{\rm ce} = 0.6$ (in the case of our study, $\omega_r \approx 0.6$, $\nu_r \approx 0.15$, and $\omega_{\rm per} \approx 0.85$; $\Delta n_e/n_e \approx 15\%$). We notice that the variation in the electron density could reach 200% and that the phenomenological collision frequency is a great influence on the relative electron-density variation. Finally, it appears it is necessary to take into account the collisional phenomenon for electron-density measurement using the resonance cone.

In conclusion, this paper points out the importance of collisions for electron-density measurements using the resonance cone. This study is based on the comparison between experimental and model results. A measurement procedure is proposed below.

(i) The signal amplitude and phase measurements are performed on the receiving antenna.

(ii) From the amplitude curves, a first estimation of the electron density, via Eq. (1), has to be done.

(iii) From the phase curves, an estimation of the phenomenological collision frequency ν is obtained by comparing the phase experimental curves and the curves obtained via Eqs. (2), (5,) and (6).

(iv) With the estimated n_e and ν values, the cone angles κ' for a collisionless plasma ($\nu = 0$) and κ for a collisional plasma are computed via the relations

$$\kappa = \arctan\left(\sqrt{-\frac{\varepsilon_\perp}{\varepsilon_\parallel}}\right), \tag{4}$$

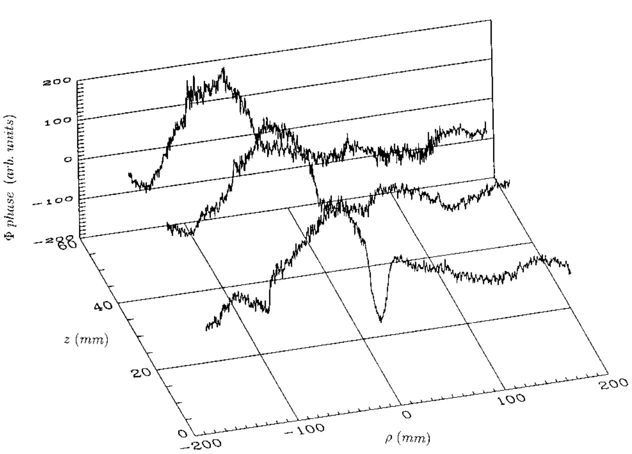

FIG. 5. Measured signal phase for different z values ($z = 10$, 20, and 30 mm), and for $B_0 = 120$ G, $f = 200$ MHz, $p_n \approx 5 \times 10^{-4}$ mbar, and discharge current $I_d = 0.46$ A.

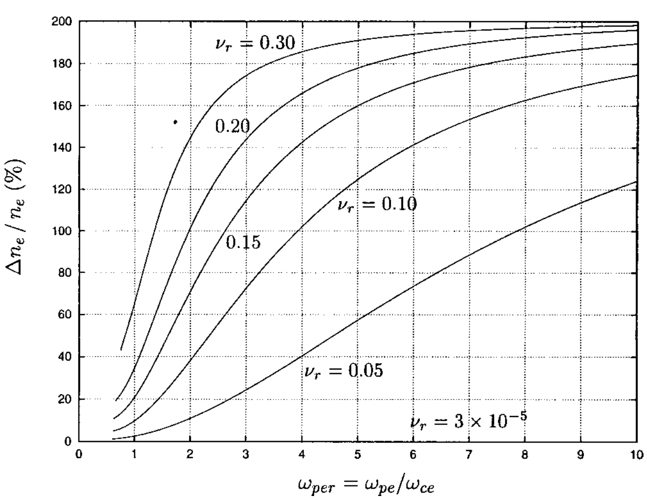

FIG. 7. Computed n_e variation ($\Delta n_e/n_e$in%) vs $\omega_{\rm per} = \omega_{\rm pe}/\omega_{\rm ce}$ for different $\nu_r = \nu/\omega_{\rm ce}$ values with $\omega_r = \omega/\omega_{\rm ce} = 0.6$.

$$\frac{\varepsilon_\perp}{\varepsilon_0}=1-\frac{\omega_{\mathrm{pe}}^2(\omega+i\nu)}{\omega[(\omega+i\nu)^2-\omega_{\mathrm{ce}}^2]}, \tag{5}$$

$$\frac{\varepsilon_\parallel}{\varepsilon_0}=1-\frac{\omega_{\mathrm{pe}}^2}{\omega(\omega+i\nu)}, \tag{6}$$

where ε_0 is the dielectric vacuum constant. We deduce that $\Delta\kappa=\kappa'-\kappa$.

(v) The correction that needs to be made to the electron density n_e is deduced via Eq. (3).

From these experimental observations, it seem difficult to obtain an electron-density error of less than 100%. It is the same order of magnitude than the electron-density measurement via a Langmuir probe without a magnetic field. But, in this case there is a magnetic field and the advantage of the resonance cone is great, since the magnetic field could greatly perturb the electron collection by the electrostatic probe.

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